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**DIFFRACTION OF A SHOCK WAVE ON A THIN WEDGE  
MOVING AT SUPERSONIC SPEED UNDER THE CONDITIONS  
OF SPORADIC WAVE INTERACTION**

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The subject of present investigation is the diffraction of a shock wave of arbitrary intensity on a thin wedge moving at a supersonic speed. The plane of the shock wave forms an almost right angle with the symmetry plane of the wedge. The interaction between the fronts is assumed sporadic. Studying the pressure perturbation along the front, a singularity of the type similar to that appearing when a weak pressure jump is diffracted on a wedge of finite opening angle with an attached shock, is discovered. This case was dealt with in [1]. The boundary value problem which is solved here using the results of [2, 3] enables us to find the pressure perturbations at the wall and along the shock front, and obtain the expression for the front in terms of elementary functions. The above problem was analyzed for the case of regular interaction in [3], where a method of generalizing the solution to the case of sporadic interaction was also suggested. The method however turned out to be impracticable.

1. A thin wedge moves through a quiescent perfect gas at a supersonic speed  $a_\infty M_\infty$  where  $a_\infty$  denotes the speed of sound in gas. The half apex angle of the wedge  $\varepsilon$  is a small parameter of the problem. At the instant  $t = 0$  the edge of the wedge encounters the front of the plane shock wave of arbitrary intensity propagating at the speed  $a_\infty M$ . The plane of the shock wave forms an angle  $\chi = \pi/2 - \delta$ , which is nearly a right angle, with the plane of symmetry of the wedge (angle  $\delta$  is of the order of  $\varepsilon$ ).

The self-similar plane motion arising at  $t > 0$  represents a perturbation in a homogeneous flow behind the shock wave.

Fig. 1a depicts the flow in the plane perpendicular to the edge of the wedge. The arc of the Mach circle the center  $E$  of which coincides with the gas particle lying at the tip of the wedge at the instant  $t = 0$  forms, together with the segments  $I'F'$  of the shock front and the part  $D'F'$  of the wall, a boundary of the region of inhomogeneous perturbations. The latter propagating at the speed  $a_1$  in the region 1 will arrive at the wavefront, but will go no further, merely causing a small distortion in the segment  $I'F'$ . The region of diffraction  $I'B'D'EF'I'$  is adjacent to two regions of homogeneous perturbation, on the left the region 5 —  $NB'D'$ , and on the right the region 2 separated from the region  $\infty$  by the front of the weak pressure jump. Diffraction of the shock wave is made more complicated by its interaction with a plane weak pressure jump caused by the supersonic flow past the wedge. Below we investigate the range of the initial values of the parameters of the problem in which this interaction cannot be regular.

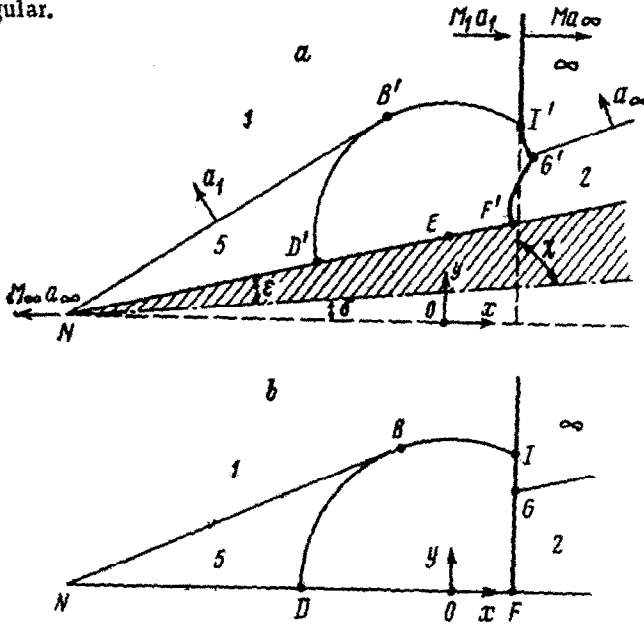


Fig. 1

It is expedient to choose a system of physical coordinates  $x', y'$  stationary with respect to the gas in region 1 so that the  $y'$ -axis coincides at the instant  $t = 0$  with the shock wavefront, while the  $x'$ -axis passes through the wedge tip  $N$  in the direction of the shockwave. The selfsimilar dimensionless coordinates can be obtained from the physical coordinates using the formulas  $x = x' / a_1 t$ , and  $y = y' / a_1 t$ .

Since the perturbations are small, we have the following boundaries of the region of diffraction (Fig. 1b) : segment  $IF$  of the unperturbed front extended to its intersection with the  $x$ -axis, segment  $FD$  of the  $x$ -axis, and the arc  $DBI$  of a unit circle with its center at the coordinate origin. The pressure  $\bar{p}$ , density  $\bar{\rho}$  and the components  $\bar{u}$  and  $\bar{v}$  of the velocity vector projected on the  $x$  and  $y$ -axis are assumed to be nearly equal, within the region described above, to the corresponding values in the region 1.

$$\bar{p} = p_1 + \epsilon \rho_1 a_1^2 p, \quad \bar{\rho} = \rho_1 + \epsilon \rho_1 \rho, \quad \bar{u} = \epsilon a_1 u, \quad \bar{v} = \epsilon a_1 v$$

Here  $\epsilon p$ ,  $\epsilon \rho$ ,  $\epsilon u$  and  $\epsilon v$  denote the dimensionless perturbations. The flow generated by the shock wave is determined by the quantities  $p_1 / p_\infty a_1 / a_\infty$ ,  $\rho_1 / \rho_\infty$ , and  $M_1$ . The coordinates of the points  $B(x_1, y_1)$ ,  $I(x_0, y_0)$ ,  $G(x_0, y_G)$ , and  $F(x_0, 0)$  are connected with the numbers  $M$  and  $M_\infty$  [3] in the following manner

(here  $\kappa$  is the polytropic exponent) :

$$\begin{aligned} x_1 &= -(M_1 + M_\infty a_\infty / a_1)^{-1}, & y_i &= \sqrt{1 - x_i^2} \quad (i=0, 1) \\ x_0 = m &= \sqrt{\frac{2 + (\kappa - 1) M^2}{2\kappa M^2 - (\kappa - 1)}}, & y_G &= \frac{a_\infty}{a_1} \frac{M + M_\infty}{\sqrt{M_\infty^2 - 1}} \end{aligned} \tag{1.1}$$

The latter formula yields the following relation connecting  $M$  and  $M_\infty$  corresponding to the given position of the point  $G$  of intersection of the fronts :

$$M_\infty = [Ma_\infty^2 / a_1^2 + y_G \sqrt{y_G^2 + (M^2 - 1)a_\infty^2 / a_1^2}] / (y_G^2 - a_\infty^2 / a_1^2)$$

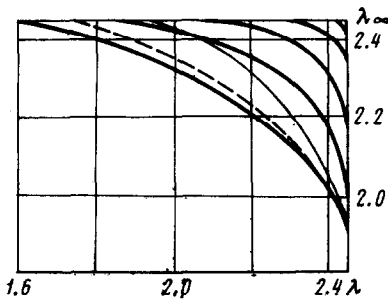


Fig. 2

The above expression is represented more conveniently in graphical form as a relation connecting the quantities  $\lambda$  and  $\lambda_\infty$  (which are the ratios of the velocities of the shock wave and the wedge to the critical speed of sound in the region  $\infty$ ). The range of values  $\lambda$  and  $\lambda_\infty$  relative to the sporadic interactions corresponds to the region lying above the lowest solid line along which  $y_G^* = y_G / y_0 = 1$  (Fig. 2). The remaining thick lines correspond to the values of  $y_G^*$  equal to 0.8, 0.6, 0.4, and 0.2 ( $\kappa = 1.4$ ).

If we fix  $\lambda$  and bring  $\lambda_\infty$  towards  $\sqrt{(\kappa + 1) / (\kappa - 1)}$ , then the point  $G$  will tend to some position at a finite distance from the wall. The point will approach the wall in an asymptotic manner, if  $\lambda, \lambda_\infty \rightarrow \sqrt{(\kappa + 1) / (\kappa - 1)}$  simultaneously.

2. We know that after passing to the spherical coordinates  $x = r \cos \theta, y = r \sin \theta$  and applying the Busemann transformation  $R = (1 - \sqrt{1 - r^2}) / r$  the function  $p$  will satisfy, within the region of perturbations, the Laplace equation. Its normal derivative on the segment  $DF$  of the boundary will be equal to zero, while along the arc segments  $IB$  and  $BD$  the pressure will be constant,  $p = 0$  on  $IB$  and  $p = p_5$  on  $BD$

$$p_5 = \frac{(M_1 + M_\infty a_\infty / a_1)^2}{\sqrt{(M_1 + M_\infty a_\infty / a_1)^2 - 1}} \left( 1 + \frac{M_1 \delta / \epsilon}{M_1 + M_\infty a_\infty / a_1} \right) \tag{2.1}$$

Here it is essential that the second derivative of the function  $p$  along the normal to the tangential discontinuity be continuous as shown by Smyrl in [3].

Linearizing the laws of conservation at the shock front the equation of which is  $x = x_0 + \epsilon f(y)$ , we arrive at the boundary values for the functions  $u, v$  and  $p$  on the segment  $IF$  of the region's boundary, the expressions for which differ from the formulas (2.1) of [1] (in which  $M_e$  must be replaced by  $M$ ) in the sign of the argument of the  $\phi$ -function only. The quantities  $h_u, h_v$  and  $h_p$  appearing there assume the form [3].

$$\begin{aligned}
 h_u &= \frac{1}{\kappa + 1} \frac{a_\infty}{a_1} \frac{2M_\infty - (\kappa - 1)M_\infty M (M + 2M_\infty)}{M^2 \sqrt{M^2 - 1}} & (2.2) \\
 h_v &= \frac{a_\infty}{a_1} M_\infty, \quad h_p = \frac{M_\infty}{\kappa + 1} \frac{p_\infty}{p_1} \frac{4M + (2M^2 - \kappa + 1)M_\infty}{\sqrt{M_\infty^2 - 1}}
 \end{aligned}$$

The expression for the perturbations in  $u$ ,  $v$  and  $p$  on the segment  $IF$  yield the boundary conditions for the function  $p$  only, and a relation used below to normalize the solution which differs from the formulas (2.2) and (2.3) of [1] in the sign preceding the  $\delta$ -function.

The Busemann transformation which converts the segment  $IF$  of the boundary into a circular arc in the plane  $\zeta = R \exp i\theta$  leads to a boundary condition for the pressure along this arc which also differs from the expression (2.5) of [1] in the sign appearing in front of the  $\delta$ -function. Superposition of the conformal mappings of the region of diffraction onto the upper half of the plane  $\omega = \xi + i\eta$  has the form [2]

$$\omega = (z^2 + \bar{z}^2) / 2, \quad z = i(\zeta \zeta_0 - 1) / (\zeta - \zeta_0), \quad \zeta = x_0 + iy_0$$

Here the wall maps onto the segment  $-1 < \xi < 1$ , the front and the Mach arc map onto the rays  $\xi > 1$  and  $\xi < -1$ , while the points  $B$  and  $G$  acquire the coordinates

$$\begin{aligned}
 \xi_B &= -[(1 - x_0 x_1)^2 + y_0^2 y_1^2] / [(1 - x_0 x_1)^2 - y_0^2 y_1^2], \quad \eta_B = 0 \\
 \xi_G &= (y_0^2 + y_G^2) / (y_0^2 - y_G^2), \quad \eta_G = 0.
 \end{aligned}$$

Taking into account the conditions on all parts of the contour we can complete the formulation of the Riemann-Hilbert problem for the analytic function  $\Gamma = \partial p / \partial \eta + i \partial p / \partial \xi$  in its closed form with the following single relation along the real axis  $\eta = 0$

$$P(\xi) \partial p / \partial \eta - Q(\xi) \partial p / \partial \xi = S \delta(\xi - \xi_G) - p_\infty \delta(\xi - \xi_B) \quad (2.3)$$

where we have

$$\begin{aligned}
 P(\xi) &= b(\xi), 1, 0; \quad Q(\xi) = 1, 0, 1 \quad \text{for } \xi > 1, -1 < \xi < 1, \xi < -1 \\
 b(\xi) &= (\gamma_1 + \gamma_2) \sqrt{\xi - 1} / (\gamma_1 \gamma_2 - \xi + 1) \\
 \gamma_{1,2} &= \sqrt{2} x_0 M \{M \pm (M^2 - 1)[M^2 + 2 / (\kappa - 1)]^{-1/2}\}
 \end{aligned}$$

The quantity  $S$  is defined by the formula (2.4) of [1] in which  $y = y_G$ . Below we shall denote  $b(\xi_G)$  simply by  $b$ .

3. In the case when the first term is absent from the right-hand side of the boundary condition (2.3), the solution was obtained by Lighthill in [2] and subsequently utilized in [3] in the course of studying the same problem with regular interaction of the wavefronts. If the whole of the right-hand side of (2.3) equals identically to zero (homogeneous problem), the solution is given by the expression

$$\Phi(\omega) = [\sqrt{\omega^2 - 1} (\gamma_1 - i\sqrt{\omega - 1})(\gamma_2 - i\sqrt{\omega - 1})]^{-1}$$

Consequently, we can write the solution of the initial inhomogeneous problem (2.3) in the form [4, 5]

$$\begin{aligned}
 \Gamma(\omega) &= \Phi(\omega)[K_1(\omega - \xi_B)^{-1} + K_2(\omega - \xi_G)^{-1} + K_3] \\
 K_1 &= (p_\infty / \pi)(\gamma_1 + \sqrt{1 - \xi_B})(\gamma_2 + \sqrt{1 - \xi_B}) \sqrt{\xi_B^2 - 1} \\
 K_2 &= (S / \pi)(\xi_G - 1 - \gamma_1 \gamma_2) \sqrt{\xi_G^2 - 1}
 \end{aligned}$$

On the real axis this solution becomes

$$\Gamma^+(\xi) = \Phi^+(\xi) \left[ \frac{K_1}{\xi - \xi_B} + \frac{K_2}{\xi - \xi_G} + K_3 \right] - ip_5 \delta(\xi - \xi_B) - \frac{S}{b+i} \delta(\xi - \xi_G)$$

Separating the imaginary part of this expression, we obtain the derivative of the function  $p$  along the wall image, and of the shock wave front

$$\frac{\partial p}{\partial \xi} = - \frac{K_1(\xi - \xi_B)^{-1} + K_2(\xi - \xi_G)^{-1} + K_3}{\sqrt{1 - \xi^2}(\gamma_1 + \sqrt{1 - \xi})(\gamma_2 + \sqrt{1 - \xi})} \quad (3.1)$$

$$\frac{\partial p}{\partial \xi} = \frac{(\gamma_1 + \gamma_2)[K_1(\xi - \xi_B)^{-1} + K_2(\xi - \xi_G)^{-1} + K_3]}{(\gamma_1^2 + \xi - 1)(\gamma_2^2 + \xi - 1)\sqrt{\xi + 1}} + \frac{S}{b^2 + 1} \delta(\xi - \xi_G) \quad (3.2)$$

$\xi > 1, \eta = 0$

The normalizing condition

$$B \int_1^{\infty} \frac{\partial p}{\partial \xi} \frac{d\xi}{y(\xi)} = h_p B - \frac{h_p B}{y_G} - \frac{a_{\infty}}{a_1} M_{\infty} - M_1 \left( 1 + \frac{\delta}{\varepsilon} \right)$$

in which the integral is understood as Cauchy principal value, yields an expression for the constant  $K_3$ , which is cumbersome and therefore not given here.

4. Integrating the expressions (3.1) and (3.2) we obtain the pressure perturbation at the wall and along the shock front (formulas (4.1) and (4.2) respectively) in terms of the elementary functions

$$p(\tau) = p_5 - \sum_{i=1}^4 c_i \operatorname{arctg} \sqrt{\frac{\gamma_i - \sqrt{2}}{\gamma_i + \sqrt{2}}} \tau - c_5 \operatorname{arctg} \frac{2y_G^* \tau}{1 - \tau^2} + \quad (4.1)$$

$$c_0 \ln \frac{\tau^2 - 2\sqrt{1 - y_G^{*2}} + 1}{\tau^2 + 2\sqrt{1 - y_G^{*2}} + 1}, \quad \tau = \sqrt{\frac{(1 - x_0)(1 + x)}{(1 + x_0)(1 - x)}}$$

$$p(y) = -\frac{1}{2} \sum_{i=1}^3 d_i \operatorname{arctg} \sqrt{\frac{(\gamma_i^2 - 2)(y_0^2 - y^2)}{2y_0^2}} - \quad (4.2)$$

$$c_0 \ln \frac{(\sqrt{y_0^2 - y_G^2} + \sqrt{y_0^2 - y^2})^2}{|y^2 - y_G^2|} - \frac{S}{b^2 + 1} \Phi(y_G - y)$$

Here

$$c_1 = \frac{4\gamma_1}{\pi \sqrt{\gamma_1^2 - 2}} \frac{\gamma_2 + \gamma_1}{\gamma_2 - \gamma_1} \left[ \frac{\sqrt{\gamma_2^2 - 2}}{\gamma_2 - \gamma_1} p_5 + s(Ay_0^2 - x_0 B) y_G^* + \frac{y_0 \gamma_2 K_0}{(\gamma_1 + \gamma_2) B} \right]$$

$$c_2 = \frac{4\gamma_2}{\pi \sqrt{\gamma_2^2 - 2}} \frac{\gamma_1 + \gamma_2}{\gamma_1 - \gamma_2} \left[ \frac{\sqrt{\gamma_1^2 - 2}}{\gamma_1 - \gamma_2} p_5 + s(Ay_0^2 - x_0 B) y_G^* + \frac{y_0 \gamma_1 K_0}{(\gamma_1 + \gamma_2) B} \right]$$

$$c_3 = \frac{2p_5}{\pi} \frac{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)}{(\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)}, \quad c_4 = \frac{2r_5}{\pi}, \quad c_5 = \frac{2}{\pi} \frac{S}{b^2 + 1}$$

$$c_0 = -(1/\pi) S b / (b^2 + 1) = (s/\pi) y_G \sqrt{y_0^2 - y_G^2}, \quad y_G^* = y_G / y_0$$

$$s = S / [(b^2 + 1)(Ay_G^2 - x_0 B)], \quad K_0 = -M_1(1 + \delta/\varepsilon) - B[h_p + S/(b^2 + 1)]/y_G$$

$$\gamma_3 = \sqrt{1 - \xi_B}, \quad \gamma_4 = -\gamma_3, \quad d_1 = c_1, \quad d_2 = c_2, \quad d_3 = c_3 - c_4,$$

$$\vartheta(y_G - y) = \begin{cases} 0, & y > y_G \\ 1, & y < y_G \end{cases}$$

It should be noted that if  $\kappa > 5/3$ , then for the values of  $M$  connected with  $\kappa$  in [6] by the necessary inequality we have  $\gamma_2^2 - 2 < 0$ . In these cases the symbol  $\arctg$  appearing in the expressions (4. 1) and (4. 2) in the terms containing the coefficients  $c_2$  and  $d_2$  must be replaced by  $\text{Arth}$ ,  $\gamma_2^2 - 2$  by  $2 - \gamma_2^2$  and  $\gamma_2 - \sqrt{2}$  by  $\sqrt{2} - \gamma_2$ .

From (4. 2) we see that the pressure perturbation along the shock wavefront consists, as in the case of diffraction which was dealt with in [1], of a smooth function, a pressure jump of magnitude  $S / (b^2 + 1)$  and a logarithmic singularity at the point  $G$ .

In Fig. 2 the thin line depicts the dependence between the characteristic parameters  $\lambda$  and  $\lambda_\infty$  of the problem under consideration, corresponding to the bifurcation for  $S = 0$  when the flow in the small neighborhood of the point  $G$  consists of homogeneous streams separated by the tangential discontinuity. The bifurcation curve corresponds to a part (lower) of the bifurcation curves characterizing the flow in the diffraction problem in [1].

Bifurcation of the second kind in which only the pressure jump is preserved, occurs when the triple point  $G$  coincides with the point  $I$  ( $y_G = y_0$ ). The lower solid line in Fig. 2 corresponds to these cases. The logarithmic term is absent from the solution also when  $y_G = 0$  (this case is depicted in [1] by the middle thick line in Fig. 2), but in the present case of a shock wave and a thin wedge, this can never be realized. The coordinate  $y_G$  tends asymptotically to zero when  $M$  and  $M_\infty$  both increase simultaneously without bounds.

The bifurcation of the third kind (only the logarithmic singularity remains) which occurs when  $y_G = \sqrt{x_0 B / A}$ , is depicted in Fig. 2 by the dashed line. It should be noted that all three curves in Fig. 2 described above emerge from the same single point.

When the wave interaction is regular ( $y_G > y_0$ ), we find that a refracted wavefront appears which touches the Mach arc and regions 3 and 4 are formed (Fig. 1 of [3]) separated by the tangential discontinuity. It can be shown that in this case

$$p_3 = p_4 = (x_0 B - Ay_G^2)S / (Ay_G^2 - x_0 B + y_G \sqrt{y_G^2 - y_0^2}) \quad (4.3)$$

The formulas obtained in [3] for the pressure at the wall and along the shock front under the regular interaction, the region lying below the lowest solid line in the  $\lambda, \lambda_\infty$ -plane corresponding to this interaction, pass smoothly as  $y_G \rightarrow y_0$  to the formulas obtained from (4. 1) and (4. 2) for  $y_G = y_0$ . The regions 3 and 4 contract to the point  $I$ , and from (4. 3) we see that  $p_3 = p_4 \rightarrow (-S)$ . Thus the solution of the problem in question and solution of the problem with regular interaction pass smoothly into each other as the wave interaction changes its character.

5. The boundary condition showing how the pressure perturbation at the front depends on its shape, can be regarded as a differential equation in  $f(y)$

$$f - yf' = (B / M_1) [p(y) - h_p \vartheta(y_G - y)] \quad (5.1)$$

in which  $p(y)$  is given by (4. 2). Its solution with the boundary condition that  $f = 0$  when  $y = y_0$  has the form

$$f(y) = \frac{B}{M_1} \left[ F(y) - \frac{c_0 y_0}{y_G} (y - y_G) \ln |y - y_G| - \right. \quad (5.2)$$

$$\left. \left( h_p + \frac{S}{b^2 + 1} \right) \frac{y_G - y}{y_G} \Phi(y_G - y) \right]$$

$$F(y) = F_1(y) - (y/y_0) F_2(y)$$

We can assume here that the function  $F_1(y)$  is given by the formula (4.2) with its last term omitted, and the denominator of the penultimate term consisting of the modulus of the difference of squares replaced by the sum of these quantities

$$F_2(y) = \sum_{i=1}^3 \frac{\sqrt{\gamma_i^2 - 2}}{2\gamma_i} d_i \operatorname{arctg} \frac{\sqrt{2}y}{\gamma_i \sqrt{y_0^2 - y^2}} +$$

$$\frac{c_0 y_0}{y_G} \ln \frac{y_0^2 (y + y_G)}{(y_G \sqrt{y_0^2 - y^2} + y \sqrt{y_0^2 - y_G^2})^2} +$$

$$\left( 1 + \frac{\delta}{\varepsilon} \right) \frac{M_1}{B} + \left( h_p + \frac{S}{b^2 + 1} \right) \frac{y_0}{y_G}$$

At the point  $J$  the perturbed front comes into contact with the vertical since  $f'(y_0) = 0$ , i. e. just as in the problem analyzed in [1], the shock wave does not undergo a break at the point of its intersection with the Mach arc.

Two last terms in (5.2) indicate the character of the singularity appearing in the shape of the front near the point  $G$  where the slope of the front can be expressed by the following approximate formula:

$$f'(y) = \frac{B}{M_1 y_G} \left[ \left( h_p + \frac{S}{b^2 + 1} \right) \Phi(y_G - y) - c_0 y_0 \ln |y - y_G| + \operatorname{const} \right]$$

In the case of bifurcation when the quantities  $S$  and  $c_0$  vanish,  $f'$  is found to have different values on different sides of the point  $G$ , i. e. the shock wavefront undergoes a break at the triple point. We also have a break in the front when the point  $G$  coincides with the point  $J$  of intersection of the shock front with the Mach arc (in this case  $c_0 = 0$ ). In all the remaining cases the values of the derivative of the front's shape and of its curvature tend to infinity as  $y \rightarrow y_G$ , and there is no break in the front. We find however, that the values of  $f'$  at two points symmetrical with respect to  $G$  differ from each other by a finite amount. This difference can be assumed to represent a jump in the value of the angle of inclination of the shock front relative to the boundaries of some neighborhood of the point  $G$  [1].

Finally, at the point  $F$  we have  $f'(0) = -(1 + \delta/\varepsilon)$ , i. e. as expected, the front is normal to the wall.

**Note.** In the case of a sporadic interaction the solution of the linear problem must have a singularity at the point  $G$ , and this aspect was not recognized by Smyrl in Sect. 8 of [3] where he attempted to generalize the solution for the regular interaction to the case discussed above. He assumed that the function  $f(y)$  can be written in the neighborhood of the point  $G$  in the form of a linear function  $f(y) = f(y_G) - (y_G - y) \delta_{1,2}/\varepsilon$ , provided that the tangents to the shock wave form at the triple point, with the  $y$ -axis, the angle  $\delta_1$  above the point  $G$  and  $\delta_2$  below  $G$ . These linear functions were then substituted into the formulas analogous to the formulas (2.1) of [1] in order to find  $u$ ,  $v$  and  $p$  separately for  $y > y_G$  and  $y < y_G$ .

The limiting values of  $u$ ,  $v$  and  $p$  thus obtained were then supposed to satisfy the conditions at the tangential discontinuity, and the conditions would lead to an expression for the break in the front  $\delta_1 - \delta_2$  in terms of  $f(y_G)$ . This would give the value of the jump in the velocity  $v$  which could be taken into account in the normalizing condition, and the generalization of the problem concluded by obtaining the quantity  $f(y_G)$  after establishing the form of the front.

This method however leads to a contradiction, since the substitution of these limiting values into the conditions at the tangential discontinuity yields an overdetermined system of two equations with a single unknown  $\delta_1 - \delta_2$  which has a solution only in the case of bifurcation.

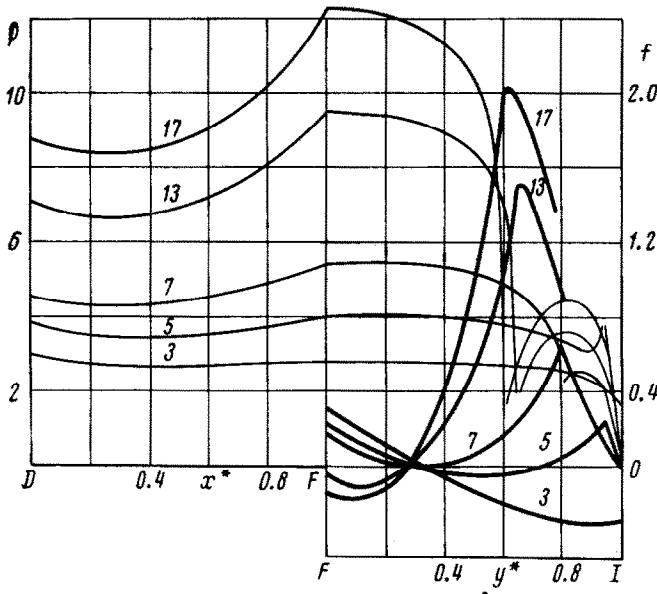


Fig. 3

The flow chart shown in Fig. 1a can be regarded as the limiting state of an asymmetric bridging configuration with two triple points  $I'$  and  $G'$  joined by the front  $I'G'$ . Analysis of the solution carried out in Sect. 4 and 5 and the computations described in Sect. 6 indicate a certain quantitative redistribution of the perturbation along the front, their considerable gradients in the neighborhood of the triple point  $G'$  and their smooth disappearance near the other triple point  $I'$ .

6. Below we show the properties of the perturbations obtained for various values of  $M_\infty$ . The dependence of the wall pressure  $p$  on  $x^* = (1+x)/(1+x_0)$  ( $0 \leq x^* \leq 1$ ) and of the pressure along the front and the shape of the front on  $y^* = y/y_0$  ( $0 \leq y^* \leq 1$ ) are shown in Fig. 3 (with  $\kappa = 1.4$ ) for  $M = 5$  and  $\delta = 0$ . The values of  $M_\infty$  are shown near the corresponding curves.

For the value of  $M_\infty = 3$  they correspond to a regular interaction ( $y_{G^*} = 1.13$ ). The shape of the front is indicated by thick lines. The value  $M_\infty = 5.54$  corresponds to a bifurcation and the jump in the pressure along the front changes its sign at  $M_\infty = 4.56$ . We see that when the shock wave is met by the wedge at large values of  $M_\infty$ , the pressure perturbations increase and their slopes become steeper with increasing  $x^*$



near the front. The maximum displacement of the latter corresponds to the neighborhood of the pressure singularity.

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**DISINTEGRATION OF AN ARBITRARY DISCONTINUITY  
IN A PERFECTLY CONDUCTING MAGNETIZABLE  
INCOMPRESSIBLE MEDIUM**

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Centered waves and strong discontinuities in a perfectly conducting magnetizable incompressible medium are investigated. It is shown that in shock waves in such medium the magnetic field tangential to the discontinuity plane and the magnetic induction increase, and the magnetic permeability decreases. In centered waves the tangential magnetic field and magnetic induction decrease. The problem of disintegration of an arbitrary discontinuity in a magnetizable perfectly conducting incompressible medium is solved by constructing diagrams in the plane of components of the tangential velocity initial shock. The diagrams make possible the determination of the combination of waves and discontinuities formed at disintegration.

Let at the initial instant of time  $t = 0$  parameters  $B_\tau$ ,  $H_\tau$ ,  $v_\tau$ , and  $T$  become discontinuous in the plane  $x = 0$ .

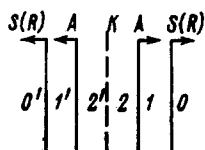


Fig. 1

If the laws of conservation are not satisfied at the discontinuity, the latter cannot exist, and it is necessary to determine the motion of medium at the following instants of time. The self-similarity of the problem implies that the motion must consist of a combination of shock waves  $S$ , centered waves  $R$ , rotational Alfvén discontinuities  $A$  and a contact